JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 24, No. 4, July–August 2001

Impulsive Feedback Control to Establish Specific Mean Orbit Elements of Spacecraft Formations

Hanspeter Schaub*
Sandia National Laboratories, Albuquerque, New Mexico 87185-1003
and
Kyle T. Alfriend†
Texas A&M University, College Station, Texas 77843-3141

An impulsive feedback control is developed to establish specific relative orbits for spacecraft formation flying. The relative orbit tracking errors are expressed in terms of mean orbit elements. The feedback control, based on Gauss's variational equations of motion, allows specific orbit elements or orbit element sets to be controlled with minimal impact on the remaining osculating orbit elements. This is advantageous when J_2-invariant orbits are to be controlled, where only the argument of perigee and mean anomaly will drift apart at equal and opposite rates. The advantage of this impulsive feedback control, compared to optimal control solutions, is that it can operate with little computational effort and in a near-optimal manner, while requiring only a marginal penalty in fuel cost. When applied to the spacecraft formation flying problem, this control could also be used to perform general orbit corrections. Formulas are developed providing accurate estimates of the sensitivities of the mean semimajor axis and mean eccentricity with respect to the osculating inclination angle. With these sensitivities, the tracking error in semimajor axis, eccentricity, and inclination angle can be canceled within one orbit.

Introduction

SPACECRAFT formation flying is an interesting and challenging topic, as is seen in Refs. 1–7. Previous formation flying control has typically been done by expressing the relative motion of a satellite to another in terms of Cartesian coordinates in the rotating local-vertical–local-horizon (LVLH) frame of one of the satellites. Reference 1 presents a continuous nonlinear feedback law to establish and maintain relative orbits of satellites. Middour[2] discusses a control strategy to maintain an along-track satellite formation where differential drag is the dominant perturbation. Melton[3] develops a time-explicit representation of the relative motion of elliptic orbits that can be used for control development. Vadali et al.[5] discusses numerically optimal initial conditions for J_2-invariant orbits. This paper will focus on expressing the relative orbit tracking error exclusively in terms of mean orbit element differences. A continuous feedback control law in terms of mean orbit elements has been presented in Ref. 4. This paper investigates an impulsive feedback control law.

The following spacecraft formation flying notations are adopted here. The satellites in the relative orbit are referred to as deputy satellites, whereas the satellite that they are orbiting about is referred to as the chief satellite. The orbit element differences $\Delta\boldsymbol{e}$ describing the relative orbit are taken from the deputy relative to the chief satellite. The Cartesian relative position vectors are computed analogously. Relative orbit errors to these desired orbit element differences are denoted as $\delta\boldsymbol{e}$. Note that it is not required here that the formation actually have a chief satellite within the formation. If none is there, then this location is simply referred to the chief because all deputy satellites are orbiting about this location. Intersatellite communication is assumed to be present to communicate their respective positions to each other.

In a gravity-dominated environment, where control is a relatively small perturbation on the overall motion, it is important to seek relative motions that are natural to the prevailing dynamics and that require little control effort to maintain. In Ref. 6, natural relative orbits to the ideal Keplerian motion are discussed. If all spacecraft involved are of equal type and build, that is, have the same ballistic coefficient, then the differential J_2 perturbation is the dominant perturbative effect experienced by the various spacecraft. For this case, the differential drag effect is negligible on the relative motion over the time period of several orbits. The Earth oblateness perturbation causes secular drifts in the ascending node Ω, the argument of perigee ω, and the mean anomaly M, along with short- and long-period oscillations in all six orbit elements. As shown in Ref. 7, it is beneficial to describe the relative orbits of a spacecraft formation in terms of mean orbit element differences $\Delta\boldsymbol{e}$, as compared to using Cartesian position and velocity vectors. By the use of mean orbit elements, the long-term behavior of the spacecraft formation is immediately evident, and short-term deviations are not considered. In trying to achieve bounded relative motion, controlling the short-term oscillation from the desired relative trajectories would be an unnecessary fuel expense if these oscillations are smaller than the required tracking accuracy.

For general formation flying, it is not possible to set up the orbit element differences between two neighboring orbits such that the three relative secular mean orbit element drifts are zero. However, it is possible to find two constraints that enforce equal ascending node and mean latitude angle rates, where the mean latitude angle is defined as the sum of the argument of perigee and the mean anomaly. Reference 7 refers to orbits satisfying these two constraints as J_2-invariant relative orbits. With these constraints on the ascending node and the mean latitude angle rates, it is possible to establish specific orbit element differences that render the resulting relative spacecraft orbit J_2 invariant. Any control maintaining these types of orbits will naturally go to zero as the desired relative orbit is asymptotically approached.

One drawback to the described orbit design methodology is that, although it does achieve equal nodal and mean latitude drift rates between various spacecraft, it is still possible for the individual arguments of perigee and mean anomalies to drift apart. Two neighboring orbits will not drift apart in the classical sense, but as the lines of perigee drift apart from their initial values, the relative orbit will either expand or contract. To counter this effect, it will be necessary to compensate periodically for both the argument of perigee and mean anomaly drift. The following impulsive control strategy was born out of the quest to find a method to correct the argument of perigee and mean anomaly while minimally impacting the remaining orbit elements. Whereas this method is attractive to compensate specific

Received 1 June 2000; revision received 6 November 2000; accepted for publication 27 November 2000.

*Research Engineer, P.O. Box 5800, Center 15211, MS-1003. Member AIAA.

†Department Head and Professor, H.R. Bright Building, Room 701, Aerospace Engineering Department. Fellow AIAA.

sets of orbit elements, it is also possible to use the method to correct for arbitrary relative orbit errors in a near-optimal manner.

To maintain desired orbit element differences, a multitude of control strategies may be employed. This paper studies a sequential impulsive algorithm that depends on mean orbit element errors to establish a specific relative orbit. Using Gauss's variational equations of motion, a firing sequence is established that allows only certain orbit element errors to be corrected during an orbit with little or no effect on the remaining orbit element differences. However, Gauss's variational equations of motion are derived for osculating orbit elements. Because specific mean orbit element differences are desired, modifications are introduced to account for the small differences between mean and osculating elements. In particular, first-order relationships between osculating orbit inclination changes and the resulting induced changes in mean semimajor axis and eccentricity are introduced. These relationships allow for a more efficient impulsive thrusting scheme to establish the desired mean orbit element differences faster. Whereas this impulsive feedback control is demonstrated and applied to the spacecraft formation flying problem, it can also be applied to the general orbit correction problem.

Problem Formulation

Gauss's variational equations are convenient to determine the effect of a control vector $\boldsymbol{u} = (u_r, u_\theta, u_h)^T$ on the osculating orbit elements, where the u_r vector component is the thrust along the orbit radial direction, u_h is along the orbit normal, and the thrust u_θ is perpendicular to the preceding two directions. Let a be the semimajor axis, e be the eccentricity, and i be the orbit inclination angle, then Gauss's variational equations of motion are given by (see Ref. 8)

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2a^2}{h}\left(e \sin f u_r + \frac{p}{r} u_\theta\right) \tag{1a}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{1}{h}\{p \sin f u_r + [(p + r)\cos f + re]u_\theta\} \tag{1b}$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{r\cos\theta}{h} u_h \tag{1c}$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{r\sin\theta}{h \sin i} u_h \tag{1d}$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{1}{he}[-p\cos f u_r + (p + r)\sin f u_\theta] - \frac{r \sin\theta \cos i}{h \sin i} u_h \tag{1e}$$

$$\frac{\mathrm{d}M}{\mathrm{d}t} = n + \frac{\eta}{he}[(p\cos f - 2re)u_r - (p + r)\sin f u_\theta] \tag{1f}$$

where p is the semilatus rectum, h is the orbit angular momentum, r is the scalar orbit radius, and θ is the true latitude angle defined as $\theta = \omega + f$. The parameter $\eta = \sqrt{(1 - e^2)}$ is another convenient eccentricity measure. These equations are written in matrix form as

$$\dot{\boldsymbol{e}}_{\mathrm{osc}} = [B(\boldsymbol{e}_{\mathrm{osc}})]\,\boldsymbol{u} \tag{2}$$

where $\boldsymbol{e}_{\mathrm{osc}}$ is the osculating orbit element vector. Note that Eqs. (1a–1f) do not necessarily show what influence the control vector will have on the mean orbit elements. Let the mean orbit element vector $\boldsymbol{e}''$ be written as a function of $\boldsymbol{e}_{\mathrm{osc}}$ as

$$\boldsymbol{e}'' = \boldsymbol{f}(\boldsymbol{e}_{\mathrm{osc}}) \tag{3}$$

where the function $\boldsymbol{f}(\,)$ could be formed from Brouwer's analytical artificial satellite theory.[9] Using the chain rule of differentiation, the mean orbit element rate equation is written as

$$\dot{\boldsymbol{e}}'' = \left[\frac{\partial \boldsymbol{e}''}{\partial \boldsymbol{e}_{\mathrm{osc}}}\right]^T \frac{\mathrm{d}\boldsymbol{e}_{\mathrm{osc}}}{\mathrm{d}t} \tag{4}$$

However, using a first-order truncation of Brouwer's transformation between osculating and mean orbit elements, it is evident that the sensitivity matrix $[\partial \boldsymbol{e}''/\partial \boldsymbol{e}_{\mathrm{osc}}]$ is essentially the identity matrix with the off-diagonal terms being of order J_2 or smaller.[4] Thus, we at first approximate Eq. (4) as

$$\dot{\boldsymbol{e}}'' \approx \dot{\boldsymbol{e}}_{\mathrm{osc}} = [B(\boldsymbol{e})]\,\boldsymbol{u} \tag{5}$$

The effect of the control vector $\boldsymbol{u}$ is assumed to have the same effect on the mean orbit elements as it has on the osculating orbit elements. The tracking accuracy penalty for imposing this initial simplification will be illustrated in the numerical simulation. The approximation in Eq. (5) will also be sufficient to establish J_2-invariant relative orbits where the required orbit element differences are of the order of J_2. The reason for this is that Eq. (5) leads to a method to control the orbit element errors. Assuming that the mean element Jacobian matrix is the identity matrix will lead to orbit correction errors on the order of J_2. As the errors tend to zero, so will the errors due to the simplified Jacobian matrix.

To refine this control approach, solving for the Jacobian matrix of the mean orbit elements would be very valuable. Because Brouwer's analytical transformation[9] between osculating and mean orbit elements is very complex, solving for the complete 6×6 sensitivity matrix is a challenging task. For the proposed impulsive control strategy, two important partial derivatives are the sensitivities of the mean semimajor axis and the mean eccentricity with respect to the osculating orbit inclination angle. Appropriate analytical expressions are derived for these partial derivatives and incorporated into the control scheme. These yield an improved convergence rate to the desired mean orbit elements compared to the solution with the Jacobian matrix being the identity matrix.

Control Strategy

When the $\mathrm{d}\Omega/\mathrm{d}t$ and $\mathrm{d}i/\mathrm{d}t$ expressions in Eq. (1) are studied, it is evident that the individual ascending node or inclination angles are adjusted best when the spacecraft passes through either the polar or the equatorial regions, respectively. However, if both an inclination angle and nodal correction are to be performed, it is more fuel efficient to perform both corrections with one impulse only. The orbit normal impulsive $\Delta v_h = u_h \Delta t$ is used to adjust both elements, as shown in Eqs. (1c) and (1d). The corresponding inclination angle and ascending node corrections are given by

$$\delta i = (r\cos\theta/h)\Delta v_h \tag{6}$$

$$\delta\Omega = (r\sin\theta/h\sin i)\Delta v_h \tag{7}$$

When Eq. (7) is divided by Eq. (6), the critical true latitude angle θ_c at which to perform this orbit normal thrusting maneuver is

$$\theta_c = \arctan(\delta\Omega \sin i/\delta i) \tag{8}$$

When Eqs. (6) and (7) are squared and summed, the required Δv_h to perform the desired inclination correction δi and ascending node correction $\delta\Omega$ is

$$\Delta v_h = (h/r)\sqrt{\delta i^2 + \delta\Omega^2 \sin i^2} \tag{9}$$

Note that applying this Δv_h affects only the orbit elements i, Ω, and ω. This cross coupling between the (i, Ω) correction and ω is the only coupling between osculating orbit element set corrections in this firing scheme. Note that although there always exist two possible critical true latitude angles θ_c from Eq. (8), only the solution corresponding to a positive Δv_h is used in this control method. Thus, (i, Ω) are corrected only at one point in the orbit.

When the Δv_h in Eq. (7) is substituted into Eq. (1e), the $\delta\Omega$ correction results in the following $\delta\omega$ change:

$$\delta\omega(\Delta v_h) = -\cos i\,\delta\Omega \tag{10}$$

This secondary effect will be taken into account when specifying the impulse required to correct the argument of perigee.

The argument of perigee and the mean anomaly are also corrected together as an orbit element pair, but with two impulsive maneuvers over one orbit. Each impulsive thrust is in the orbit radial direction only and is applied at both the orbit perigee and apogee. Let Δv_{r_p} be the radial impulse applied at perigee and Δv_{r_a} be the impulse at apogee. Computed over one orbit, and taking into account that an ascending node correction $\delta\Omega$ could be occurring (which causes an additional change in ω), the Δv_{r_p} and Δv_{r_a} impulses cause the following osculating orbit element changes:

$$\delta\omega = (1/he)\left[-p\left(\Delta v_{r_p} - \Delta v_{r_a}\right) - \delta\Omega\cos i\right] \tag{11}$$

$$\delta M = (\eta/he)\left[(p - 2r_p e)\Delta v_{r_p} - (p + 2r_a e)\Delta v_{r_a}\right] \tag{12}$$

To solve these two equations for the radial Δv, the following identities are useful:

$$p - 2r_p e = p[(1-e)/(1+e)] \tag{13a}$$

$$p - 2r_a e = p[(1+e)/(1-e)] \tag{13b}$$

along with $h/p = na/\eta$. Substituting these expressions into Eqs. (11) and (12), we find

$$\Delta v_{r_p} - \Delta v_{r_a} = -(\delta\omega + \delta\Omega\cos i)(nae/\eta) \tag{14}$$

$$(1-e)^2\Delta v_{r_p} - (1+e)^2\Delta v_{r_a} = nae\delta M \tag{15}$$

Solving these two equations for the required radial impulses to achieve a desired $\delta\omega$ and δM, we find

$$\Delta v_{r_p} = -(na/4)\{[(1+e)^2/\eta](\delta\omega + \delta\Omega\cos i) + \delta M\} \tag{16}$$

$$\Delta v_{r_a} = (na/4)\{[(1-e)^2/\eta](\delta\omega + \delta\Omega\cos i) + \delta M\} \tag{17}$$

Note that if a $\delta\Omega$ correction is performed during this orbit, then its effect is immediately taken into account in the preceding two equations.

The argument of perigee and mean anomaly corrections, provided by Eqs. (16) and (17), are convenient to compensate for the natural secular drift in these orbit elements that will occur with the J_2-invariant orbit presented in Ref. 7. Only ω and M of the six orbit elements will not have an equal relative drift rate, but rather their sum will. This relative drift difference is not very large, but depending on the tolerances of the relative orbit it will require compensation periodically. Furthermore, the smaller the eccentricity of the orbit, the smaller the effect that relative drift of ω and M will have on the orbit geometry. However, Eqs. (16) and (17) provide an impulsive control method that is able to readjust directly the argument of perigee and mean anomaly while minimally affecting the other osculating orbit elements.

The remaining two orbit elements to be corrected are the semimajor axis a and the eccentricity e. As is the case with the argument of perigee and mean anomaly corrections, the semimajor axis and eccentricity are adjusted together through two impulsive maneuvers over one orbit. However, these impulsive thrusts are fired in the transverse u_θ direction. One impulsive correction Δv_{θ_p} is fired at perigee, and the other impulse Δv_{θ_a} is fired at apogee. With this firing sequence, a and e are adjusted efficiently and without disturbing the other osculating orbit elements. From Eq. (1), the a and e corrections over one orbit are

$$\delta a = (2a^2/h)\left[(p/r_p)\Delta v_{\theta_p} + (p/r_a)\Delta v_{\theta_a}\right] \tag{18}$$

$$\delta e = (1/h)\left[(p + r_p + r_p e)\Delta v_{\theta_p} + (-p - r_a + r_a e)\Delta v_{\theta_a}\right] \tag{19}$$

Note that in deriving Eqs. (18) and (19) it is assumed that the orbit corrections δa and δe are relatively small. Otherwise a and e could not be held constant during the two maneuvers. To solve these two equations for the tangential Δv, the following identities are used:

$$p + r_p + r_p e = 2p \tag{20}$$

$$-p - r_a + r_a e = -2p \tag{21}$$

Equations (18) and (19) are now rewritten as

$$(1+e)\Delta v_{\theta_p} + (1-e)\Delta v_{\theta_a} = (h^2/2a^2)\delta a \tag{22}$$

$$\Delta v_{\theta_p} - \Delta v_{\theta_a} = (h/2p)\delta e \tag{23}$$

Using $h/a = na\eta$, with $\eta = \sqrt{(1-e^2)}$, the required tangential impulses are found to be

$$\Delta v_{\theta_p} = (na\eta/4)[\delta a/a + \delta e/(1+e)] \tag{24}$$

$$\Delta v_{\theta_a} = (na\eta/4)[\delta a/a - \delta e/(1-e)] \tag{25}$$

Note that in both the ω, M and a, e corrections, the sequence of impulsive maneuvers over an orbit is irrelevant. The first maneuver may occur at either perigee or apogee.

To implement these impulsive Δv, the mean orbit element errors are established at some arbitrary point in the orbit and are then held constant during the orbit while appropriate Δv are applied, as discussed earlier. This impulsive firing scheme assumes that all of the mean orbit element errors will remain constant over an orbit. If the a, e, and i elements do not satisfy the J_2-invariant conditions in Ref. 7, then Ω, ω, and M will experience some J_2-induced secular relative drift. However, this drift is typically relatively small over an orbit. The impulsive feedback control will correct, or at least substantially reduce, any remaining mean orbit element errors during the following orbit. The exception is if the deputy semimajor axis is substantionally different from that of the chief. In this case, the different orbit periods will cause the mean anomaly to exhibit substantial relative drift over one orbit. In this case, it cannot be assumed that δM is constant over an orbit. Thus, the ω, M corrections do not begin until the second orbit. Doing this allows the a, e, and i variables to be corrected during the first orbit, which will set the orbit periods equal between deputy and chief satellite. During further orbits, any remaining relative mean anomaly errors will remain constant over an orbit. If the ω, M corrections are applied during the first orbit with a large semimajor axis error present, then the impulsive feedback control law still corrects the relative orbit. However, the fuel cost typically increases because incorrect ω, M corrections are performed during the first orbit.

Note that the relative orbits are only corrected at a few points during the orbit. Thus, the satellites are coasting for significant portions of the orbit where the small perturbations are moving the spacecraft from their desired relative orbits. The magnitude of these drifts depends on how the relative orbit is designed. When a J_2-invariant relative orbit is used, the dominant gravitational perturbation is compensated for with the desired relative orbit geometry. Mission relative orbit tracking tolerances will dictate whether or not the coast periods of the impulsive feedback control will yield acceptable tracking performance.

Because it is advantageous to describe the relative orbit in terms of orbit element differences of the deputy satellite relative to the chief satellite, this impulsive firing sequence is a convenient method to correct orbit errors from the desired orbit element differences. If only one or two elements are to be adjusted, then this control solution is essentially optimal. If several orbit elements are to be corrected, then preliminary studies have shown this method still to yield a near-optimal solution with a fuel cost increase of only a few percent over the multi-impulse optimal solution. The advantage of this method is that through its simplicity and low computational overhead, it lends itself well to implementation in an autonomous manner. Little ground support would be required for a cluster of spacecraft to maintain their formation as long as they are able to sense their inertial orbits themselves. This could be achieved through global positioning system (GPS) measurements[10] or direct line-of-sight measurements between the various satellites.[11] Feeding back mean orbit element errors has the benefit that any short-period oscillations are ignored.

Further, it is convenient to be able to adjust only certain orbit elements, leaving the remaining elements virtually untouched. For relative orbits designed using the approach outlined in Ref. 7, the resulting relative orbit will be J_2-invariant in an angular sense. This means that the neighboring orbits will have equal nodal and mean latitude drift rates. However, the argument of perigee and mean anomaly will still drift apart at equal and opposite rates. The consequence of this drift is that the relative orbit will go through cycles of symmetrically growing and shrinking as the chief satellite completes one orbit. This effect is more noticeable for satellite clusters with larger eccentricities. For a cluster with nominally zero eccentricity, having the argument of perigee and mean anomaly grow apart at equal and opposite rates has no effect on the overall relative orbit geometry. Furthermore, this impulsive firing scheme could also be used as the initial conditions for an optimizer solving for the true minimum-fuel orbit correction. Often indirect optimizing methods are sensitive to initial conditions, and the proposed impulsive feedback law could provide a reasonable initial guess as to the structure of the optimal control solution.

Selected Mean Orbit Element Sensitivities

An important issue not considered so far is that mean, not osculating, orbit elements are to be controlled with the spacecraft formation flying. As a first approximation, it is feasible to assume that the $\partial \boldsymbol{e}/\partial \boldsymbol{e}_{\text{osc}}$ matrix, given in Eq. (4), is a 6×6 identity matrix. However, for the tight tolerances required with formation flying, the effects of $\partial \boldsymbol{e}/\partial \boldsymbol{e}_{\text{osc}}$ must also be considered. For example, whereas the inclination angle and ascending node correction should not affect any other osculating elements (except for the argument of perigee), it does affect particular mean orbit elements.

When a numerical study is performed, it is evident that adjusting the osculating orbit inclination correction does have a noticeable effect on the mean semimajor axis and eccentricity. However, adjusting the osculating semimajor axis and eccentricity, as shown in Eqs. (24) and (25), has a negligible effect on the remaining mean elements.

This section develops algebraic formulas for the sensitivities of the mean semimajor axis and the eccentricity with respect to the osculating inclination angle. With these formulas, it is possible to predict the effect that the inclination angle correction will have on these selected mean orbit elements and to incorporate this information when computing the required a and e corrections. During the first orbit, these formulas will result in near-perfect cancellations of tracking errors in a, e, and i.

When a first-order truncation of the Brouwer artificial satellite theory[9] is used, the mean semimajor axis a'' is given by

$$a'' = a - a(J_2/2)\left(r_e^2/a^2\right)\{(3\cos i^2 - 1)[(a/r')^3 - 1/\eta^3] + 3(1-\cos^2 i)(a/r')^3 \cos(2\omega' + 2f')\} \tag{26}$$

where r_e is the Earth radius, f is the true anomaly, and r is the current orbit radius. When Brouwer's notation is used, double-primed variables are the mean orbit elements, single-primed variables have the long-period terms removed and unprimed variables are the osculating parameters. Note that Eq. (26) involves both unprimed and single-primed variables. This makes the precise development of $\partial a''/\partial i$ very challenging. As suggested in Refs. 9 and 12, the computational accuracy of the mean element computation is equivalent if the unprimed and primed ω and f are interchanged. The mean semimajor axis is now expressed as

$$a'' = a - a(J_2/2)\left(r_e^2/a^2\right)\{(3\cos i^2 - 1)[(a/r)^3 - (1/\eta^3)\} + 3(1-\cos^2 i)(a/r)^3 \cos(2\omega + 2f)\} \tag{27}$$

Taking the partial derivative of Eq. (27), we find

$$\frac{\partial a''}{\partial i} = -\frac{3}{2} J_2 \sin(2i) \frac{r_e^2}{a}\left\{\frac{1}{\eta^3} + \left(\frac{a}{r}\right)^3 [\cos(2\omega + 2f) - 1]\right\} \tag{28}$$

Thus, for a given orbit inclination angle correction δi, the corresponding change in mean semimajor axis $\delta a''$ is given by

$$\delta a'' = \frac{\partial a''}{\partial i}\delta i \tag{29}$$

If only an inclination correction is performed, then the impulse is applied at either $\theta = \omega + f = 0$ or 180 deg. Equation (29) is then reduced to the simpler form

$$\delta a'' = -J_2 \frac{3}{2}\frac{r_e^2}{a}\frac{\sin(2i)}{\eta^3}\delta i \tag{30}$$

From Brouwer's artificial satellite theory,[9] a first-order approximation of the mapping between the mean and osculating eccentricity is given by

$$e'' = e - \delta_1 e - (J_2/4)(\eta^2/e)\left(r_e^2/a^2\right)\{(3\cos^2 i - 1)[(a/r')^3 - 1/\eta^3] + 3(1-\cos^2 i)[(a/r')^3 - 1/\eta^4] - (e/\eta^4)(1-\cos^2 i) \times [3\cos(2\omega' + f') + \cos(2\omega' + 3f')]\} \tag{31}$$

with the variable $\delta_1 e$ given by

$$\delta_1 e = (J_2/16)\left(r_e^2/a^2\right)(e/\eta^2) \times \left[1 - 11\cos^2 i - 40\cos^4 i(1 - 5\cos^2)^{-1}\right]\cos(2\omega) \tag{32}$$

When the same assumptions as were done when developing the partial derivative of a'' are made, the partial derivative of e'' with respect to the osculating inclination angle i is

$$\frac{\partial e''}{\partial i} = -\frac{J_2}{4}\frac{r_e^2}{a^2}\frac{\sin(2i)}{\eta^2}\left(\frac{e}{4}\left[11 + \frac{80\cos^2 i}{1 - 5\cos^2 i} + \frac{200\cos^4 i}{(1-5\cos^2 i)^2}\right] \times \cos(2\omega) + \frac{3}{e\eta^4}\left\{\left[\left(\frac{a}{r}\right)^3 - \frac{1}{\eta^3}\right] + \left[\left(\frac{a}{r}\right)^3 - \frac{1}{\eta^4}\right] \times \cos(2\omega + 2f)\right\} - 3\cos(2\omega + f) - \cos(2\omega + 3f)\right) \tag{33}$$

The change in mean eccentricity $\delta e''$ due to a correction in osculating inclination δi is then given by

$$\delta e'' = \frac{\partial e''}{\partial i}\delta i \tag{34}$$

Again, if only inclination angle corrections are performed individually, that is, $\theta = 0$ or 180 deg, Eq. (34) reduces to

$$\delta e'' = -\frac{J_2}{16}\frac{r_e^2}{a^2}\frac{\sin(2i)}{\eta^2}\left\{\left[11 + 80\cos^2 i(1 - 5\cos^2 i)^{-1} + 200\cos^4 i(1-5\cos^2 i)^{-2}\right] e\cos(2\omega) - 12\frac{1-\eta}{e} \mp 16\cos\omega\right\}\delta i \tag{35}$$

where the minus sign is used if $\theta = 0$ deg and the plus sign is used if $\theta = 180$ deg.

When initializing the mean orbit element tracking errors that are to be corrected during the following orbit, the $\delta a''$ and $\delta e''$ are now added to the actual mean orbit element tracking errors to account for the effect of correcting the inclination angle. With this adjustment, numerical simulations illustrate that the mean a, e, and i errors can be canceled within one orbit. The remaining mean orbit element sensitivities are to be derived in future work. For the given control problem, obtaining the a'' and e'' sensitivities with respect to i was required to improve the convergence rate.

Numerical Simulations

The following numerical simulation establishes a desired J_2-invariant orbit by employing the impulsive control scheme presented in this paper. The chief mean orbit elements and the desired deputy mean orbit element differences are shown in Tables 1 and 2. The relative orbit has a prescribed inclination angle difference of 0.006 deg, while the semimajor axis and eccentricity are adjusted to compensate for this. The initial mean orbit element errors of the deputy satellite are $\delta a'' = -100$ m, $\delta i'' = 0.05$ deg, and $\delta\Omega'' = -0.01$ deg.

Table 1 Chief mean orbit elements

Element	Value	Units
a	7555	km
e	0.05	
i	48	deg
Ω	20.0	deg
ω	10.0	deg
M	120.0	deg

Table 2 Desired relative orbit element differences

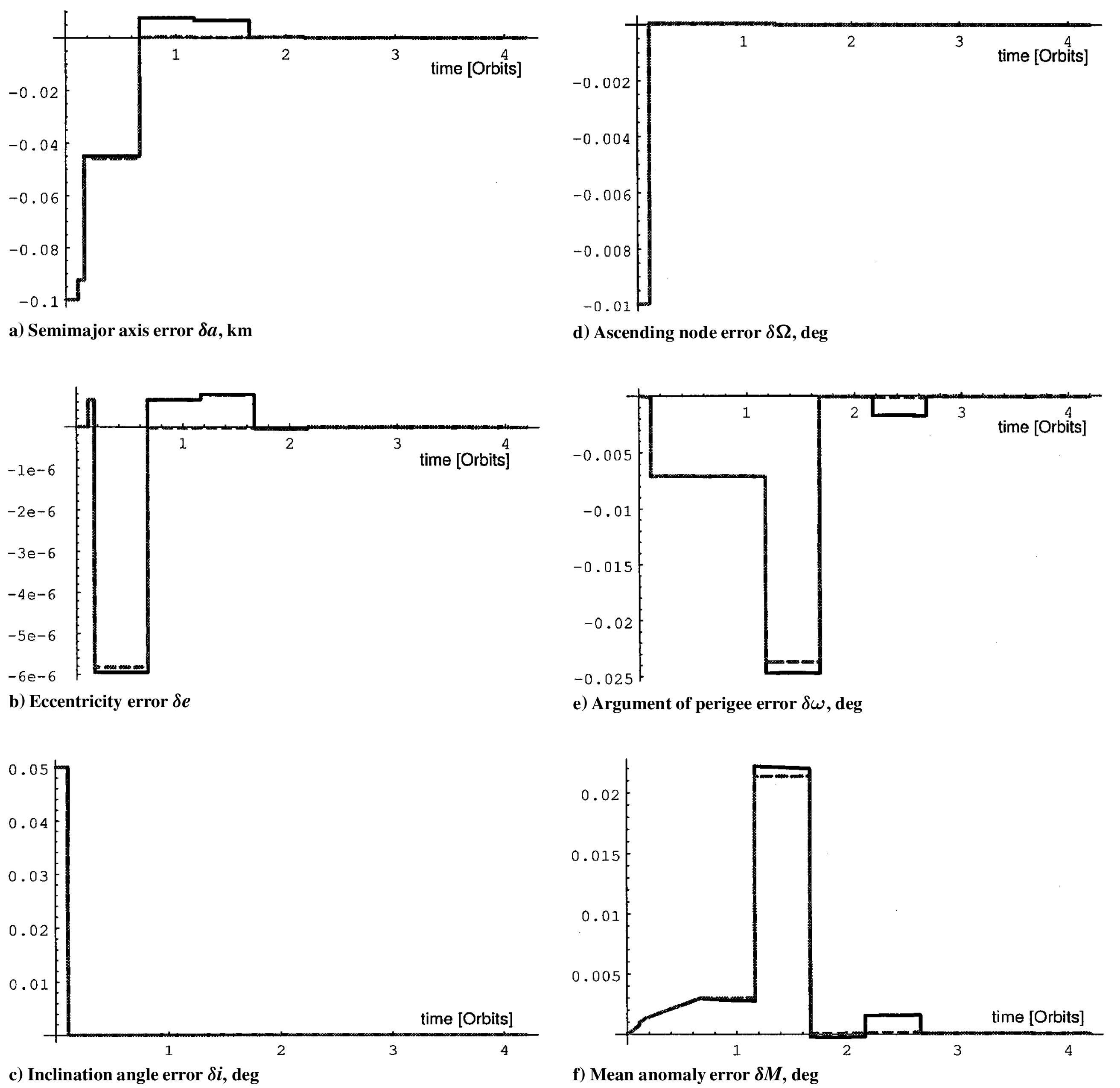
Difference	Value	Units
Δa	−0.00192995	km
Δe	0.000576727	
δi	0.006	deg
$\Delta\Omega$	0.0	deg
$\Delta\omega$	0.0	deg
ΔM	0.0	deg

Figures 1a–1f show two test runs. The nonlinear equations of motion

$$\ddot{\boldsymbol{r}} + \mu \boldsymbol{r} = \boldsymbol{f}(\boldsymbol{r}, J_2, J_3, J_4, J_5) \tag{36}$$

are integrated for each spacecraft including the gravitational zonal harmonics up to fifth order. This allows for a numerical verification that the predictions based on Gauss's and Brouwer's theories are valid. In case 1 (shown as a dashed line) the impulsive control scheme is employed without making use of the partial derivatives of a'' and e'' with respect to the inclination angle. In case 2, the same control scheme is used with the addition that if inclination angle corrections are performed, then their effects on a'' and e'' are included. During the first controlled orbit corrections on semimajor axis, eccentricity, inclination angle, and ascending node are attempted. Again, the purpose is to first match the orbit periods and then attempt to correct the mean anomaly errors.

Case 1 is able to reduce the initial tracking errors in a, e, and i substantially during the first orbit. However, it is clear that when the osculating inclination angle is corrected, the mean semimajor axis and eccentricity are also affected. When the osculating orbit elements during the first orbit are studied, no change in the latter



a) Semimajor axis error δa, km

d) Ascending node error $\delta\Omega$, deg

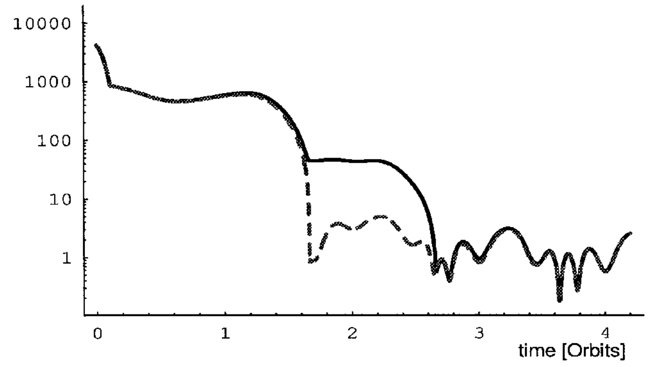
b) Eccentricity error δe

e) Argument of perigee error $\delta\omega$, deg

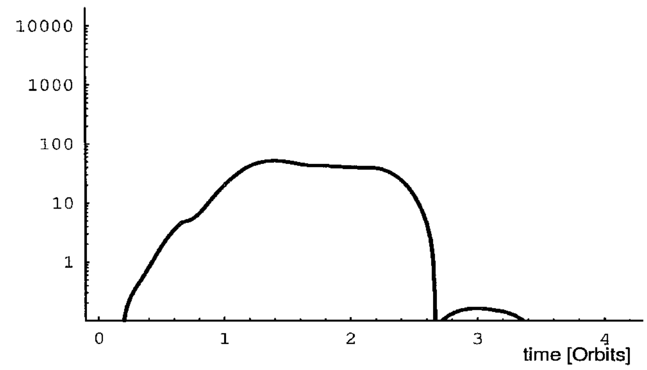
c) Inclination angle error δi, deg

f) Mean anomaly error δM, deg

Fig. 1 Mean orbit element tracking errors (- - - -, case 1, and ——, case 2).



a) Scalar Cartesian tracking error, meters



b) Tracking error difference between case 1 and case 2, meters

Fig. 2 Relative orbit tracking errors in Cartesian coordinates.

three orbit elements was observed as i is corrected, as is predicted according to Gauss's variation equations. For the given orbit elements, the effect of changing the osculating inclination by -0.05 deg is roughly 7.315 m on the mean a. In case 2, the sensitivities of the mean a and e with respect to the osculating i are utilized. The result is that after the first orbit the mean elements a, e, i, and Ω are at the desired values. This verifies that the simplifications performed in deriving the two mean element sensitivities still resulted in a good prediction of these partial derivatives.

From the second controlled orbit onward all six orbit elements are corrected per orbit. After the second orbit, the orbit element tracking errors for case 2 are essentially zero. Case 1 requires an extra orbit iteration to cancel out the remaining small tracking errors.

Figure 2a shows the scalar, radial tracking error of the relative orbit for both cases 1 and 2, with case 1 again the dashed line. Both cases converge to the same level of tracking accuracy of about 1 m. This is the same level of accuracy as was achieved with the feedback control laws in Ref. 4. The reason these control laws do not reduce the tracking error to zero is due to using a first-order truncation of Brouwer's artificial satellite theory[9] when translating between osculating and mean orbit elements. Figure 2b shows the difference in tracking errors between cases 1 and 2. Note that the difference during the second orbit is too small to be seen in Fig. 2a. Note that the tracking error of case 2 has reached its lower limit before the end of the second orbit. The tracking error of case 1 does not reach its lower limit before the third orbit is completed.

The relative orbit trajectory, as seen in the rotating LVLH frame, is shown for either case in Fig. 3. The differences between the trajectories of cases 1 or 2 are too small to be visible at the scale shown. The reference relative orbit is shown as the black curve, with the location of the chief satellite shown as a point inside the relative orbit. The point where the first inclination angle and ascending node correction occurs is plainly visible here as a an abrupt change in the relative orbit plane. After three orbits, the actual relative orbit does approach the desired trajectory. Note that the impulsive feedback control law does a very good job in converging to the desired relative orbit. Even within one orbit period, the actual relative orbit is very close to the desired relative orbit.

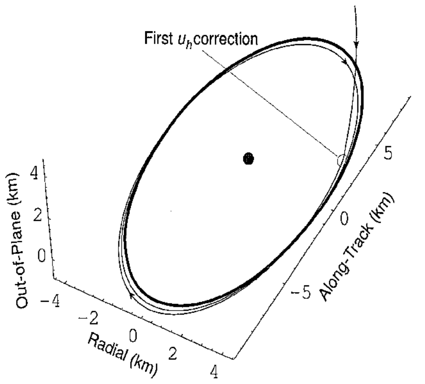


Fig. 3 Relative orbit as seen in rotating LVLH reference frame, kilometers.

The total Δv consumed with case 1 is 6.4550 m/s. The Δv for case 2 is reduced slightly to 6.4184 m/s. Even though typically case 2 provided a better fuel economy than case 1, it was possible to set up the initial orbit element tracking errors such that case 1 had a slightly lower fuel consumption. As a comparison, either case has a lower fuel consumption than what was found for the feedback laws in Ref. 4, where the mean element feedback control law required 7.584 m/s for the same initial errors. The Cartesian coordinate feedback law in Ref. 4 required 7.428 m/s. The Δv for a two-impulse optimal orbit correction required 6.24 m/s. This means that, for the given initial conditions, the presented impulsive feedback control law commanded only a 3% Δv penalty compared to the fuel-optimal solution.

The impulsive control law presented is not necessarily intended to replace precomputed, fuel-optimized maneuvers. If the time and computational effort is available, fuel-optimal maneuvers should be employed. What the impulsive feedback control does provide is a simple logic with which to do orbit corrections. Because these corrections are near optimal, it is feasible that a spacecraft would be able to perform relative motion station keeping without the extensive ground support required for doing optimal trajectories.

Conclusions

An impulsive feedback control strategy in terms of mean orbit element differences is presented to maintain a cluster of formation flying spacecraft. The control compares the orbit element differences to predefined values and adjusts various orbit elements at particular regions of the orbit. The elements a, e (semimajor axis, eccentricity); ω, M (argument of periapsis, mean anomaly); and i, Ω (inclination, right ascension of the ascending node) are adjusted as pairs. The orbit element corrections are designed such that they only marginally influence the remaining osculating orbit elements. Because mean orbit elements are to be tracked, the sensitivities of the mean semimajor axis and eccentricity with respect to the osculating inclination angle are presented. With these formulas, it is possible to essentially cancel all mean a, e, i, and Ω errors within the first orbit, whereas the ω, M are corrected during the second orbit. The impulsive feedback control law requires little computational effort compared to fuel-optimal solutions, yet achieves the orbit correction in a near fuel-optimal manner. Because of its simplicity, this control technique lends itself to autonomous relative orbit station keeping without extensive ground support, as long as the individual satellites are able to determine their orbits themselves.

Acknowledgments

This research was supported by the Air Force Office of Scientific Research (AFOSR) under Grant F49620-99-1-0075; the authors are pleased to acknowledge the coordination of M. Q. Jacobs at AFOSR.

References

[1]Kapila, V., Sparks, A. G., Buffington, J. M., and Yan, Q., "Spacecraft Formation Flying: Dynamics and Control," *Proceedings of the American Control Conference*, IEEE Publications, Piscataway, NJ, 1999, pp. 4137–4141.

[2]Middour, J. W., "Along Track Formationkeeping for Satellites with Low Eccentricity," *Journal of Astronautical Sciences*, Vol. 41, No. 1, 1993, pp. 19–33.

[3]Melton, R. G., "Time-Explicit Representation of Relative Motion Between Elliptical Orbits," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 4, 2000, pp. 604–610.

[4]Schaub, H., Vadali, S. R., and Alfriend, K. T., "Spacecraft Formation Flying Control Using Mean Orbit Elements," *Journal of Astronautical Sciences*, Vol. 48, No. 1, 2000, pp. 69–87.

[5]Vadali, S. R., Schaub, H., and Alfiend, K. T., "Initial Conditions and Fuel-Optimal Control for Formation Flying of Satellites," AIAA Paper 99-4265, Aug. 1999.

[6]Sedwick, R., Miller, D., and Kong, E., "Mitigation of Differential Perturbations in Clusters of Formation Flying Satellites," American Astronautical Society, AAS Paper 99-124, Feb. 1999.

[7]Schaub, H., and Alfriend, K. T., "J_2 Invariant Reference Orbits for Spacecraft Formations," NASA Flight Mechanics Symposium, Paper 11, May 1999; also *Journal of Celestial Mechanics* (to be published).

[8]Battin, R. H., *An Introduction to the Mathematics and Methods of Astrodynamics*, AIAA Education Series, AIAA, New York, 1987, p. 488.

[9]Brouwer, D., "Solution of the Problem of Artificial Satellite Theory Without Drag," *Astronomical Journal*, Vol. 64, No. 1274, 1959, pp. 378–397.

[10]Chu, Q., and Woerkom, P., "GPS for Low-Cost Attitude Determination—A Review of Concepts, In-Flight Experiences, and Current Developments," *Acta Astronautica*, Vol. 41, No. 4–10, 1997, pp. 421–433.

[11]Psiaki, M. L., "Autonomous Orbit Determination for Two Spacecraft from Relative Position Measurements," *Journal of Guidance, Control, and Dynamics*, Vol. 22, No. 2, 1999, pp. 305–312.

[12]Lyddane, R. H., "Small Eccentricities or Inclinations in the Brouwer Theory of the Artificial Satellite," *Astronomical Journal*, Vol. 68, No. 8, 1963, pp. 555–558.